IDENTIFYING MATHEMATICAL MISUNDERSTANDINGS IN THE
STUDENTS AND THEIR TEACHER

Submitted by Dr. Peter Farrell
Bachelor of Applied Science (Honours)
Graduate Diploma in Education (Primary)
Master of Applied Science
Doctor of Education

A thesis submitted in partial fulfilment of the requirements for the postgraduate certificate
in primary mathematics teaching

Faculty of Education
RMIT University
Melbourne VICTORIA 3001
Australia

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APPROVALS

‘Except where reference is made in the text of the thesis, this thesis contains no material published elsewhere or extracted in whole or in part from a thesis submitted for the award of any other degree or diploma.

No other person’s work has been used without due acknowledgement in the main text of the thesis.

The thesis has not been submitted for the award of any degree or diploma in any other tertiary institution.

All research procedures reported in the thesis were approved by the relevant Ethics or Safety Committee or authorized officer as appropriate.’

Peter Farrell
ABSTRACT

This is a ‘professional’ thesis written in light of Donald Schön reflective practitioner model. It investigates the outcomes arising from a change in delivery model around mathematics teaching in a very small rural school. I conclude that Schön’s reflection on reflection-in-practice template is an effective methodology for combining my current professional knowledge with new academic thinking. I conclude that Mathletics is a useful way to deliver content to a multi-age classroom however teacher support must amount to more than procedural fixes if student misunderstandings around mathematics are to be avoided in the future.
ACKNOWLEDGEMENTS

I would like to acknowledge the assistance of the DEECD in paying 80% of the cost of the course. I would like to thank Professor Seimon and her team at RMIT University for their willingness to share knowledge and expertise. I always appreciated the assistance provided by my coach Sharyn Levy and the discussions I had with my group of adult learners. I would like to acknowledge the thinking of Donald Schön who has strongly influenced the layout of this ‘professional’ thesis. As is usual I must acknowledge the forbearance of my wife, Della, and although this is a shorter thesis than some others I have completed, it will be arguably more useful for the students in my classroom.
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CHAPTER 1 - INTRODUCTION

Introduction

This chapter describes the context of the research project, provides a problem statement, describes the aim of the research, and the outline of the research approach. I provide a clarification of terms and an overview of the thesis.

Context of the Research Project

Zeerust Primary School is a very small, one-class primary school, situated in a rural district in the Northeast of the state of Victoria. It has served its district since 1928 and now its parent clientele tend to be a mixture of professionals, small business owners and wage earners who work in the nearby centre of Shepparton. In 2010 it had 16 students and this year there were 15. With respect to mathematics teaching in 2009 the school began to use two new approaches, the first, the Numeracy Fluency Assessment was a specific analytical tool mandated by the regional office to be used to determine those number concepts the child had mastered and what concepts were still to be learnt. The second, Mathletics, was an on-line maths program that was used at Zeerust to deliver ability-appropriate content to all students simultaneously.

Problem Statement

The problem of practice with respect to teaching mathematics at Zeerust Primary School was the wide range of students that had to be taught. Selecting appropriate topics, tasks and activities was always problematical. Discussing the situation with other teaching-principals in a similar situation to my own suggested that Mathletics might have some utility as a means to deliver appropriate subject content to all my students. With computer: student ratios of better than 1:1 at my school every student could be on line simultaneously and complete the coursework at their own pace. In this context, teacher became tutor, responding to student requests for help, or through observation of the student’s lack of progress as indicated by the program itself. It was noted that tutor responses tended to be ‘procedural’ rather than ‘conceptual’ and, until the present study; no systematic attempt was made to determine the effectiveness of the change to using Mathletics.
**Aim of the Research**

The aim of this research is to investigate the unexpected outcomes arising from the use of an on-line mathematics program in a very small rural school.

**Outline of the Research Approach**

For the present study a ‘professional’ as opposed to an ‘academic’ research approach was adopted. Having completed an Ed.D., a professional doctorate, following the methodology reminiscent of a Ph.D., I have strongly held views about how rigorous professional learning should look. Unlike academic study where the literature is examined for gaps, conundrums and truths, and from this, a research question is generated and subsequently investigated. A professional study begins with the problem and looks to solve that problem. In fact, the professional researcher should be willing to manipulate the variables in order to achieve the desired outcome whereas the academic attempts to control them. Since completing my doctorate of education I have written a reflective piece about that piece of work and subsequently read the work of Donald Schön and the present study is quite deliberately framed in light of reflective professional practice.

**Clarification of Terms**

*Table 1. Terms used in the thesis*

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMHO</td>
<td>In my honest opinion</td>
</tr>
<tr>
<td>DEECD</td>
<td>The Victorian state department for education and early childhood development. Occasionally referred to in the present study as the department. The department funded 80% of the cost of the PGCPMT.</td>
</tr>
<tr>
<td>RMIT University</td>
<td>Royal Melbourne Institute of Technology University. The organisation running the PGCPMT.</td>
</tr>
<tr>
<td>PGCPMT</td>
<td>Postgraduate Certificate in Primary Mathematics Teaching. This is a course of professional learning aimed at experienced teachers looking to upgrade their teaching and leadership of mathematics.</td>
</tr>
</tbody>
</table>

**Overview of Thesis**

The thesis is divided into six chapters:
• **Chapter 1 – Introduction:** This is the current chapter where I set the scene and introduce the structure of the thesis. I begin with a problem statement; the context for the research; the aim of the research; an outline of the research approach; a clarification of terms used in the thesis (Table 1), and; finally I provided this overview of the structure of the entire thesis.

• **Chapter 2 – Research Design:** I begin by briefly describing the basic premise of the study and my research aim. I list my research question and who my sample group is. Next I explain my research methodology, which is embedded in the reflective practice paradigm. Next I explain the data gathering process and finish the chapter with an outline of the approach used to analyse the data.

• **Chapter 3 – Reflection-in-action:** In this chapter I begin by presenting each reflection-in-action on each ‘ah ha!’ moment shared by my students about their learning. Next the aggregated biographical data is presented followed by a summary of the misunderstandings presented, the way the student attempted to overcome the problem and, whether s/he was successful.

• **Chapter 4 – Common Misconceptions in Learning Mathematics:** This is the literature review chapter and arises from the findings of the results chapter which suggest some of the students in the present study are affected in their learning of mathematics because of the misunderstandings they have about certain mathematical ideas.

• **Chapter 5 – Reflection on reflection-in-action:** This is the chapter where I present my reflections on reflections-in-action in light of my reading of the literature.

• **Chapter 6 – Discussion and Conclusions:** I begin by examining the utility of the approach used to gather data. Then I address the research questions: asking so? So what? And now what?

**Conclusion**

In this chapter the research problem has been defined and the reader has been introduced to the scope of the study and how it fits into a wider context of professional
research. A detailed overview of the structure of the thesis and clarification of some of the terms used has been provided. In the next Chapter I look at research design.
CHAPTER 2 – RESEARCH DESIGN

Introduction

In this Chapter, I concern myself with the research design adopted for the present study. It is not a literature review although some literature is referred to; especially that of Donald Schön, the champion of reflective practice in the professions. It begins with the basic premise that professional as opposed to academic knowledge is legitimate. We then outline our research aim, which is concerned reflecting on the use of the Mathletics program at my school. A discussion of Schön’s ideas introduces the reader to my research paradigm. The research question, sample group, and gathering and analysing of data are then described.

Basic Premise

In 1987 Donald Schön published an influential book entitled, ‘Educating the Reflective Practitioner’ where he argues that skilful practice needs professionals with the competence and artistry to find solutions for real world problems which are not, ‘well formed structures [but] messy, indeterminate situations (p.4)’. Schön (1987, p. 25) uses the term, ‘knowing-in-action’, which is the idea that a ‘person’ reveals intelligent action though ‘spontaneous skilful execution’ of that particular action. Schön (1987) notes that the ‘person’ is, ‘characteristically unable to make it [the action] verbally explicit (p.25)’.

Schön’s thesis is that through active observation of, and reflection upon, our actions, the practitioner forms constructions about their professional world. These constructions embrace ideas, values, skills and knowledge (of which only a small part is derived from research p.13). However, the constructs are dynamic and, through the process of rigorous reflection, continuously tested, and always subject to revision. It is a constructivist view, one concerned with phenomenology. With respect to the revision process, Schön (1987) refers to the dual ideas of ‘reflection-in-action (p.26)’ and ‘reflection on reflection-in-action (p.309)’. Reflection-in-action might be likened to thinking on your feet or thinking-in-action. The ability to effectively think on your feet requires the capacity to call upon knowledge, experience and ideas at need. The reflection on reflection-in-action is a term developed by Schön (1987, p. 309) to describe how professional knowledge can be legitimised (in a university setting). The reflective practice requires that some time pass
before an event be thought about in a deep and meaningful way. Schön (1987, p. 342) suggests that this process of learning might be best suited to experienced professionals engaged in a continuing their education (as is the case for this author). In this scenario Schön (1987, p. 311) the role of the coach becomes paramount and the coaches ability depends on artistry as a coach rather than skill as a lecturer or attainments as a scholar. The reflective process requires the coach help surface the knowing-in-action of the student whilst at the same time linking the student with appropriate and helpful research-based theories.

The basic premise of the present study is that the systematic gathering of evidence and reflection-in-action on that evidence followed by subsequent reflection on reflection-in-action is a legitimate professional process and the professional learning from these activities are valid.

**Research Aim**

The aim of this research is to investigate the unexpected outcomes arising from the use of an on-line mathematics program in a very small rural school.

**Research paradigm**

Schön (1987, p. 70) discusses the differences between academic and professional experimentation and makes the point that the rigor strived for in hypothesis testing is not possible under the conditions prevailing in professional practice. Schön (1987) claims that practitioners will set up an experiment with a desired outcome in mind and will manipulate the variables, if they can, to make their hypothesis come true (p.73). As well as hypothesis testing Schön (1987) also describes other forms of experimentation carried out by practitioners like exploratory experimentation and move-testing experimentation. Exploratory experimentation describes an approach where an, ‘action is undertaken only to see what follows, without the accompanying predictions or expectations (p.70)’. Move-testing is, ‘an action undertaken with an end in mind … one either gets the intended consequence or does not (p.71)’. The introduction of the Mathletics program to the students in the present study was a move-testing experiment in that it was anticipated that the
program could deliver appropriate mathematical curriculum simultaneously to a multi-age group of students in a single-class school.

**Research question**

One question will be investigated: What are the common mathematical misconceptions surfaced by the students in the present study using Mathletics?

**The Sample Group**

The sample group came from students in the author’s own mathematics classroom. The class is multi-age and multi-grade. The students were aged from five to 12 years old where there was one prep, two grade twos, three grade threes, five grade fours, one grade five and three grade six students. Each student was undertaking an on-line Mathletics course at their own pace. Five students were working two years above their grade level, three students were working on content 12 months ahead of their grade. Six students were at their expected grade. One student, a grade six, was working on grade five material. Not all students are discussed and some are discussed more than once. All the names used are identifiably male or female.

The students discussed in this study were those who were stumped by a particular concept in their respective course. The on-line program, Mathletics, visually shows student progress through the content via the use of coloured bar-graphs. The student must successfully complete the same task, made up of 10 questions, three times before s/he is considered ‘perfect’. Perfect indicates a score of 80% or better and the bar is full length and gold in colour. Where the student has successfully completed the task twice. The bar is full length, yellow in colour and labelled ‘good job’. Where the student has successfully completed the task once. The bar is full length and coloured blue. If the student should fail, a red bar one third of the length of the ones described above is indicated. A failure can come about if the student attempts all 10 questions and gets less than 80% correct or, starts the task but fails to finish. If a student fails twice, a red bar, two-thirds the length of a ‘perfect’ bar, is shown. It is at this point a misconception is suspected. Sometimes the student will use the on-line help to find an answer, other times they might use a peer, and occasionally they just ask me and we work on the problem together.
The data gathering process

Finding the 2/3 red bars was a matter of continuously roving around the class. On some occasions I would spot the red bar myself and on others, students would identify their own, or someone else’s to me. At first, the ‘ah ha!’ moment, was seen just as a sign of student success; and so it is. But over time, the successful mastery of a mathematics concept came to be valued by the students as a sign of progress. One unanticipated consequence of the process was that the students would try much harder at a particular task and it became more difficult to find students struggling to master an idea.

A digital camcorder in my personal phone proved to be an extremely effective and efficient way to capture what the students were saying. I could do this during class time. As soon as possible after class I referred to the recording and wrote my reflection-in-action piece where I immediately commented upon what I had observed and what the student had said. For some students they would share their ‘ah ha!’ moment with the rest of the class with the assistance of a data projector. To do this they would work through the task that had stumped them and how they eventually solved it. For some students this was too public and I recorded their thinking one-on-one during the session.

The data analysis process

The written ‘reflection-in-action’ was the first piece of analysis and these are presented in chronological order. From these writings a table summarising biographical details, learning problems, approaches and outcomes was created and themes were ascertained.

Conclusion

The research design chapter has introduced the reader to the methodology adopted in the present study. The methodology adopted is a move-testing experiment where reflection-in-action records the consequences for students of using an on-line mathematics program in a multi-age classroom. In Chapter 3 that follows I share my reflections-in-action.
CHAPTER 3 – REFLECTION-IN-ACTION

Introduction
The results here begin with the presentation of my written reflection-in-actions and these are followed by an analysis of the sample group which is broken down into biographical and other data. The other data is concerned with identifying the mathematical misconception of the student, what strategy was adopted to overcome the misunderstanding and whether that was successful.

The reflections
The written reflection-in-action pieces are presented here in chronological order. The first reflection, Anne, was collected on February 11th and the last, Colin, was recorded on June 30th. Their names have been changed.

11 Feb 2011
“OUR first attempt at maths communication occurred yesterday. Anne is a grade five student working through grade seven Mathletics content. Anne likes a challenge but not too much. Anne was stumped by a directed number task in the integer section of her course. In a nutshell she had to decide what number was represented by a mark on a number line. All was well until the scale changed from say 0, 10, 20 to 0, -5, -10 and so on. Her 'ah ha!' moment was figuring out the half-way point between -5 and -10 was -7.5 and then the task was easy. Anne worked through this herself. Can't wait for the next story which is Euler's formula?”

14 Feb 2011
“Beatrice is a grade six student working on the grade seven course and her 'ah ha!' moment came about through understanding Euler's formula. Euler's formula is $V + F - E = 2$ and provides a way to work out Vertices, Faces and Edges in a polyhedral shape. Beatrice was presented with a representation of a triangular prism and asked to work out $F + 6$ Vertices = 9 Edges + 2. Using prior knowledge of three-dimensional shapes Beatrice was able to count the 5 Faces. Similarly, the next shape, a cube was summarily dealt with by counting the vertices and writing this into the formula. At this point Beatrice had no idea
she was dealing with a mathematical formula. The problem came when Beatrice was presented with a complex polygon where she was told $X \text{ Faces} + 12 \text{ Edges} = 30 \text{ Vertices} + 2$. Beatrice was stumped. There was no way she could reliably count the faces in the diagrammatic representation. Beatrice's 'ah ha!' moment was recognising the equals sign the middle of the formula and understanding that she had to make both sides of the equal sign the same value. From this point on it was relatively simple arithmetic to determine the answer. Reflecting on this suggests this might be a real good place to start Beatrice on algebra.”

16 Feb 2011

“Florence is a grade three student working through grade three content. Each year Mathletics archives student results and so I have to record how far through a course a student got. In 2010 Florence completed 86% of her grade three course - not enough IMPO to go up to the grade four course in 2011. Florence was stumped by the precise wording used in a harder probability proposition. Her 'ah ha!' moment came about when she recognised the significance of 'will not' in the wording of the problem. A later observation on the importance of language in mathematical text was very apparent during practicing for the numeracy NAPLAN when a common problem was mis-reading questions.”

23 Feb 2011

“Carol is a grade six girl working on grade five content. The task, which stumped Carol, was to do with probability and offered up scenarios to which she could respond probably, possibly, even, unlikely and impossible. Carol indicated she was ready to share her 'ah ha!' moment and so we set up the task on the IWB and she began the 1st of 10 questions around this concept. It turned out that she had not experienced an, 'ah ha!' moment after all. What she had done was to use the help facility in the software to work out the correct answer - she did not 'own' the concept herself.”

24 Feb 2011

“Deanne a grade two student working on the grade three course was attempting something easier (i.e. grade two level) concerned with measurement of area. Interestingly
she struggled to successfully complete the task in front of the class. Deanne was using non-mathematical methods and her results were not promising. With Carol [see below], we stopped the show and tell. With Deanne we continued and took her through the process of counting grid squares, Deanne could count by twos and threes. Once Deanne bought into the counting approach her frequency of correct answers increased significantly.”

3 Mar 2011

“Carol came back to explain her 'ah ha!' moment around probability again. This time she able to answer the problems in the task but she was unable to articulate her 'ah ha!' moment. What had happened was she had got better at 'remembering' the correct answers from the help function in the program and working with a couple of other kids. What was apparent is that she got correct answers by dint of persistence and continuous practice. Carol did not 'own' the concept.”

28 Mar 2011

“Andrew is a grade four student completing the year 4 course. In general, Andrew is a good problem solver but he does have low literacy. Andrew's 'claytons' 'ah ha! moment came from a task concerned with identifying acute, obtuse and 90 degree right angles. Apart from the right angle Andrew was unable to differentiate between acute and obtuse and then - he figured it out. He followed a rule. If the angle was sharp then the answer was always the choice button on the left. If the angle was blunt then it was the choice button on the right. Andrew was absolutely right because this is how the software presented each problem. However, when presented with a handwritten angle, Andrew could not correctly identify what it was. His rule worked but only in the Mathletics environment.”

27 Apr 2011

“Andrew was working on a task that required him to identify the largest decimal number. However, Andrew was stumped. As I observed him complete the 10 questions again it became apparent that Andrew was attracted to numbers with more digits. When I asked him to consider the importance of place value he still did not get it. His ah ha! moment came when I began to talk specifically about there being 2 tenths as opposed to 1
tenth. This worked. This observation is interesting because in teaching Andrew earlier in Term 1 he was found to have a very good grasp of decimal numbers on a number line.”

18 May 2011

“Brian is a grade four student working on the grade four course. He was stumped by a task that required him to round off decimal numbers to the nearest whole number. As I observed work on the 10 questions it was pretty evident that he was making educated guesses only to find they did not necessarily hold true for the next example. I drew a number line for Brian on the board going from number x to number y. I identified the halfway point as 0.5. It didn't help as much as I hoped - he still didn't get it. I then explained to him that numbers above x.5 were rounded up to y while numbers below x.5 were rounded down to x. This helped. Although numbers like x.05 did give him pause, until I asked him to consider the place value. 'Ah Ha!'”

30 Jun 2011

“Colin a grade four student working on grade four content was solving problems based around pie graphs. All was well until he needed to calculate 10, 20 and 30%. Colin was unable to guess or even intuit his way to a sensible answer and asked for my help. Working out percentages is a mini lesson I have given to many students over the years. We moved to the whiteboard and started with 100, 50 and 25% using halves and quarters as ways in. Colin followed this easily. For the percentages I showed how the number could move one decimal place to the right and give the right answer. Colin readily understood this and we were able to extend calculating 10% to working out 20 and 30% as well as 5%.”

Summary of biographical data

Of the 15 students in the class eight students contributed to the data; two students (Carol and Andrew) contributed twice (Table 2). The students ranged from Deanne in grade two to Beatrice and Carol each in grade six. Colin, Brian and Andrew were in grade four while Florence and Anne were in grades three and five respectively (Table 2). With respect to the mathematical content this group were working on it ranged from Anne and Beatrice
each working on grade seven content to Deanne and Florence each working on grade three content. Colin, Brian and Andrew were working on grade four content and Carol was working on grade five content (Table 2).

Table 2. Summary of biographical data, misunderstandings, approaches and outcomes

<table>
<thead>
<tr>
<th>Name</th>
<th>School grade</th>
<th>Mathletics grade</th>
<th>Misunderstanding identified by reflection-in-action</th>
<th>Learning approach to overcome misunderstanding</th>
<th>Successful or unsuccessful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andrew (a)</td>
<td>4</td>
<td>4</td>
<td>Place value</td>
<td>Teacher demonstration</td>
<td>Yes</td>
</tr>
<tr>
<td>Andrew (b)</td>
<td>4</td>
<td>4</td>
<td>An artefact</td>
<td>Self-solved</td>
<td>No</td>
</tr>
<tr>
<td>Anne</td>
<td>5</td>
<td>7</td>
<td>Place value</td>
<td>Self-solved</td>
<td>Yes</td>
</tr>
<tr>
<td>Beatrice</td>
<td>6</td>
<td>7</td>
<td>Relational thinking</td>
<td>Self-solved</td>
<td>Yes</td>
</tr>
<tr>
<td>Brian</td>
<td>4</td>
<td>4</td>
<td>Place value</td>
<td>Teacher demonstration</td>
<td>Yes</td>
</tr>
<tr>
<td>Carol</td>
<td>6</td>
<td>5</td>
<td>Weak literacy</td>
<td>On-line help</td>
<td>No</td>
</tr>
<tr>
<td>Carol *</td>
<td>6</td>
<td>5</td>
<td>Rote learning</td>
<td>Self-solved</td>
<td>No</td>
</tr>
<tr>
<td>Colin</td>
<td>4</td>
<td>4</td>
<td>Place value</td>
<td>Teacher demonstration</td>
<td>Yes</td>
</tr>
<tr>
<td>Deanne</td>
<td>2</td>
<td>3</td>
<td>Non-mathematical thinking</td>
<td>Teacher demonstration</td>
<td>Yes</td>
</tr>
<tr>
<td>Florence</td>
<td>3</td>
<td>3</td>
<td>Weak literacy</td>
<td>Self-solved</td>
<td>Yes</td>
</tr>
</tbody>
</table>

* The two events are related.

Summary of misunderstandings, approaches and outcomes

Three problems with learning mathematics were identified (Table 2). Six students (Colin, Beatrice, Brian, Andrew and Anne) reported problems with their learning relating to a common misunderstanding in mathematics. The misunderstandings related to place value (Colin, Brian, Andrew and Anne), and relational thinking (Beatrice). Two students, Carol and Florence, demonstrated weak literacy on one occasion each and on a second occasion,
Carol relied upon rote learning to solve her initial problem. Andrew recorded a problem that arose as an artefact of the Mathletics environment. Deanne, used non-mathematical thinking, which was hard to understand. There were four and five occasions respectively where the students either learnt from a teacher demonstration or mastered the concept themselves (Table 2). These were all successful. There was one occasion where the on-line help was used but this was not successful.

The most common misconception related to place value and all the grade four students and the one grade five student indicated a problem in this area. The four grade four students, each studying grade 4 content, required teacher demonstrations to reach their ‘ah ha!’ moment whilst the grade five student (Anne), working on grade seven material, was able to self-solve her problem.

**Conclusions**

In this chapter I have presented the ‘reflection-in-action’ reflections for eight students in a class of 15. What is apparent that while some problems do relate to mathematical misunderstandings, which are presented in more detail in the next chapter, some issues appear unrelated to mathematics.
CHAPTER 4 – COMMON MISUNDERSTANDINGS IN LEARNING MATHEMATICS

Introduction

After collecting and analysing my data I now turn to the literature, the subject of which, has been suggested by my reflections-in-action in Chapter 3. I want my Chapter 5 discussion, where I present my reflection on reflection-in-action, to be conducted in light of theory and more widely accepted truths. It is my hope that this reflective process will lead to improved teaching and learning outcomes for my students.

The aim of this research is to investigate the unexpected outcomes arising from the use of an on-line mathematics program in a very small rural school. With respect to the common misconceptions in mathematics a number of noteworthy authors could have been chosen but here I use the work provided by Professor Dianne Seimon (2006). There are three principle reasons for this:

• Professor Seimon’s ideas have been adopted by the DEECD (Department of Education and Early Childhood Development) and this is the system in which the present study is posited.

• Professor Seimon is leading the RMIT University team in delivering the PGCPMT to DEECD employees, and this is the course this author is completing; and

• As a reflective professional I do not intend to get into esoteric arguments about the advantages of one framework over another.

As stated much of what follows is sourced from the departmental website which was last updated in October 2009 and is the output of Professor Seimon’s work. The site is primarily about the assessment for common misunderstandings and provides tools for teachers to facilitate assessment along with advice about what the results indicate about that student’s understanding. The material I have made most use of are the key indicators for each common misunderstanding, which are directly quoted and a paraphrasing of the reasons, provided by Professor Seimon, as to why a student may have a difficulty in this area of mathematics. In addition to the website, two text-books, which have been part of my personal collection for many years are referred to. The second edition of ‘Teaching Primary
Mathematics’ and, ‘Math Matters: Understanding the math you teach’. Both textbooks are concerned with mathematics at the primary school level and so those misunderstandings, recognised as occurring in secondary school (Level 5 and 6), are examined in light of two journal articles. Howe, Nunes and Bryant published their paper about proportional reasoning in 2010 and this is referred to in my discussion of Level 5 misunderstandings. Warren & Cooper (2009) wrote their paper about equivalence and algebraic thinking and this is used in the discussion of Level 6 misunderstandings. A third journal article by Willis, written in 2002, is concerned with aspects of counting and is referred to in the discussions of Level 1 and Level 2 misunderstandings.

**Level 1 - Trusting the count**

‘A key indicator of the extent to which students have developed mental objects for the numbers 0 to 10 is the extent to which they can recognise collections of these numbers without counting, that is, they can subitise (Seimon 2006)’. Subitising is the ability to make sense of how many without counting for example, to instantly recognise (3) as three (Chapin & Johnson, 2000, p. 11). Willis (2002) notes that some children can chant out the numbers but cannot count (p.118) whilst others can subitise very small collections say (3) but cannot do (4) because they cannot count (p.120). Willis goes on to state that larger collections usually have to be organised in a recognisable pattern before being subitised correctly.

Students who are unable to count may not realise that counting is an accepted strategy to work out how many, or may not have made the connection between the numbers spoken and the objects counted (cardinality). Rectifying these deficiencies are the first task of formal instruction (Booker, Bond, Briggs & Davey 1998, p. 59). Another issue for the student might be an inability to recognise, read or write number names and numerals. Booker *et al.* (1998, p. 60) makes a strong argument that children learning to count must have opportunities to see and use numbers as names (four), objects (4) and symbols (4). It is also possible that poor organisation of the objects might also lead to some items not being counted at all or counted more than once. Chapin & Johnson (2000, p. 11) suggest that students need to count a great many things early in their learning, and that strategies for coordinating counting need to be demonstrated to students.
Chapin & Johnson (2000, p. 47) identify six common counting strategies and these include: counting all, counting down from, counting down to, counting up from, counting on from first, and counting on from larger. Chapin & Johnson (2000, p. 47) suggest that while students are capable of creating novel counting techniques do not assume that students are able to simply memorise these techniques. Counting strategies are conceptually based and each strategy makes different demands on the user. In all instances there are varying degrees of the student ‘trusting the count’ (Seimon 2006).

**Level 2 – Place-value**

‘A key indicator of the extent to which students have developed a sound basis for place-value is the extent to which they can efficiently count large collections and confidently make, name, record, compare, order, sequence, count forwards and backwards in place-value parts, and rename 2 and 3 digit numbers in terms of their parts (Seimon 2006).’

Booker et al. (1998) devote a whole chapter (chapter 3) to numeration opening with a quote that, ‘beyond simple counting, all calculations with whole numbers and decimal fractions make use of place value (p.53)’ and later state, ‘naming [and renaming] numbers is the most fundamental concept needed in numeration (p.54)’. Booker et al. (1998) say that numeration is a crucial topic in its own right as well as being a ‘foundation for many other procedures (p.55)’. Indeed an inability to understand place value is a very common cause of misunderstanding in mathematics.

Students with weak place-value knowledge may have an inability to trust the count or even count efficiently using two, five or ten as units of the count. They may have inadequate part-part-whole knowledge for the numbers 0 to 10, in fact they not be able to make much sense of numbers beyond 10 for example, 14 is 10 and four more, or that the number six in sixty four represents six 10s (Seimon 2006). Willis (2002, p. 123) notes that some children have a sense of twoness or fiveness etc but this may or may not translate to an understanding of skip counting. Some children may not realise that skip counting is simply a more efficient way of counting large collections than by ones.
**Level 3 - Multiplicative thinking**

‘A key indicator of the extent to which students have developed a broader range of ideas to support multiplicative thinking is the extent to which they manipulate both the size of the group and the number of groups to meet specific needs (e.g., instead of committing 6 eights to memory in a meaningless or rote way, recognise that this can be thought of as 5 eights and 1 more eight, or 3 eights doubled) (Seimon 2005).’

Chapin & Johnson (2000, p. 53) indicate that in earlier times multiplication and division focussed on the step-by-step process of the calculation to an eventual correct answer. Little focus was given to conceptual understanding but Chapin & Johnson (2000, p. 60) argue that both procedural understanding and conceptual understanding are equally important and go further to say that multiplication and division are the, ‘visible parts of an enormous conceptual iceberg that also includes ratio, rational numbers, linear function, dimensional analysis and vector space’. Chapin & Johnson (2000, p. 60) put the view that moving from counting to additive to multiplicative thinking occurs slowly.

Students experiencing difficulties with multiplicative thinking may not trust the count or see different countable units in their own right (e.g. view 6 items as 1 six (“a six”) rather than 6 ones). They may have poorly developed or non-existent mental strategies for addition and subtraction and demonstrate an over-reliance on physical models to solve simple multiplication problems. The student may have had limited exposure to alternative models of multiplication (Seimon 2006).

**Level 4 – Partitioning**

‘Key indicators of the extent to which students have developed an understanding of fractions and decimals is the extent to which they can construct their own fraction models and diagrams, and name, record, compare, order, sequence, and rename, common and decimal fractions (Seimon 2006).’

Chapin & Johnson (2000) devote one chapter each to the subjects of fractions and decimals, chapter five and six respectively. In their introduction to fractions Chapin & Johnson (2000, p. 73) state that, ‘this topic has caused more trouble for elementary and middle school students than any other area of mathematics’. Fractions are rational numbers but can have multiple meanings and interpretations for example, fractions can be parts of
wholes or parts of sets, fractions can be calculated from the division of two numbers, a fraction can represent a ratio between two quantities and, fractions can also indicate scale (Chapin & Johnson 2000, p. 74).

In chapter six, Chapin & Johnson (2000, p. 98) introduce decimals as an extension of place value knowledge and suggest teachers experience little trouble with respect to the performance of computations however, they claim conceptual teacher understanding of decimals is such that, ‘many of us find it hard to articulate even the most fundamental ideas’. It should be noted that unlike the Booker text, which is Australian, the Chapin & Johnson text is American. The initial problem starts with the fact that decimal numbers are real numbers and while it is possible to write some decimals as fractions it is not universal (Chapin & Johnson 2000, p. 98). Next is the misunderstanding about the purpose of the decimal point which is to indicate the positions of the ones place and subsequently all other places. The base 10 numeration system relies upon the face and place value of a digit.

Seimon (2006) stated students might experience difficulties with fractions because they view the denominator in the same way as the numerator (i.e., as a count or ‘how many’ number, rather than an indication of ‘how much’). The student may have had a limited exposure to practical experiences that show what happens as the number of parts are increased, and how fractional parts are named. The student may have had the groups of only idea for multiplication and division or little or no access to strategies that support the construction of appropriate fraction representations. Booker et al. (1998, p. 96) makes a strong case for the teaching of language and the use of physical models to represent fractional ideas.

**Level 5 - Proportional reasoning**

At level 5 we move from common misunderstanding in primary mathematics into the realm of misconception occurring in secondary school mathematics. However, I believe it is always useful to know where students are headed even if it is not our responsibility or task to teach at that level.

‘A key indicator of the extent to which students have developed a broader range of ideas to support proportional reasoning is the extent to which they use multiplicative strategies such as partitioning to solve problems involving simple proportion (e.g., find for
1 or a common composite unit such as 3 in a comparison involving 6 and 9, then multiply as appropriate). Another indicator is the extent to which students can work meaningfully with multiple representations of proportional relationships (Seimon 2006).

Howe et al. (2010, p. 391) state that by the end of elementary schooling in the USA and the UK the emphasis on rational number in mathematics teaching increases by an order of magnitude. Misunderstandings about rational numbers resonate in the USA whilst ratios and percentages are of particular concern to UK policymakers and so it is not very surprising that the factors adversely affecting Australian students around proportional thinking are many. Seimon (2006) created a long list that included:

- Students with a limited range of ideas for multiplication and division.
- Students with a limited experience of manipulating real and rational numbers.
- A reliance on rule-based procedures to rename fractions.
- A limited understanding of what it means to multiply or divide a quantity by a rational number, and
- A limited understanding of ratio.

‘The chain that runs from rational number through proportional reasoning to achievement in science carries implications for practice. For one thing it suggests that rational number should be introduced in a fashion that anticipates other parts of the chain (Howe et al., 2010, p. 393)’.

Howe et al. (2010, p. 393) suggest that, ‘many key science notions in science revolve around intensive quantities which combine direct and indirect proportions’. ‘In fact, all intensive quantities can be represented using ratios, whilst a subset can be represented using fractions, decimals and percentages (p.393).

**Level 6 - Generalising, equivalence, number properties and patterns, and the use of algebraic text**

‘A key indicator of the extent to which students are ready to engage with these curricula expectations is their capacity to deal with equivalent forms of expressions, recognise and describe number properties and patterns, and work with the complexities of algebraic text (Seimon 2006)’
Warren & Cooper (2009, p. 90) conclude that a student’s understanding of mathematics in this domain is directly impacted by the effectiveness of the teaching the student has experienced. They say (p.91) there is no one model to bring students to abstract thinking but an effective model does clearly show the underlying mathematical idea and allows for some extending of that idea. An effective teaching sequence allows the student to work from the static to the dynamic to the abstract model while at the same time the models become less overt and more mentally embedded in the student’s mind but the basis for the model is in the real world (p.91).

Seimon (2006) notes that the difficulties experienced by students in making the transition from arithmetic to algebra may be due to:

- Their poor understanding of the meaning of symbols or the conventions used in algebraic expressions.
- Their limited understanding of arithmetic, numeration and computation.
- Their slavish reliance on procedural rules rather than conceptual understanding.
- Their lack of experience in communicating mathematical relationships in words and/or translating relationships described in words into symbolic expressions.
- Their limited access to multiplicative thinking and proportional reasoning.

Seimon’s list suggests that the misunderstandings around generalised mathematics are set in place long before the student enters the grade 9 mathematics classroom.

**Conclusion**

The common misunderstanding in mathematics identified by the DEECD website and based upon the work of Professor Seimon, provided the framework for the literature review above. The framework covers both primary and secondary schooling but I would like to adapt the metaphor by Howe et al. (2010) about mathematics being a chain of ideas to my conclusion. What Howe et al. (2010) suggested was that it was important for teachers to anticipate the different parts of the chain when teaching a particular [mathematical] concept. With this in mind it would be educational to follow some of the misunderstandings from Level 6 through to Level 1 if possible. Misunderstandings with multiplicative thinking have far-reaching consequences for students with adverse learning outcomes being seen at Levels 6, 5, 4 and 3. Another serious weakness, showing up in Levels 6, 5 and 3, is an
over-reliance on rules and a concomitant weakness in conceptual understanding. Proportional misunderstandings at Level 6 first arise in Level 4 where misunderstandings occur around rational numbers. Weaknesses in additional thinking at Level 3 can be traced to problems with counting in Level 1.

In this chapter I have explored the mathematical misunderstandings that occur in Victorian schools from Level 1 to 6. At each Level a critical misunderstanding about mathematics has been identified along with the behaviours demonstrated by students with these misunderstandings. These misunderstandings will play a part in my reflections on reflections-in-action at Chapter Five.
CHAPTER 5 – REFLECTION ON REFLECTION-IN-ACTION

Introduction

This chapter could be likened to a second results chapter. The reflection on reflection-in-action chapter begins with a discussion of each student and includes my initial diagnosis of their misunderstanding and my subsequent diagnosis after reviewing the literature. Some reflection on my prescription to overcome the problem is also included. The reflections are broadly divided into mathematical and non-mathematical misunderstandings.

Discussion arising out of the study related to the common misunderstandings in mathematics

Colin

My reflection-in-action indicated I thought Colin’s misunderstanding was about his failure to understand percentages related to his incomprehension of place value. Reflection on reflection-in-action shows proportional reasoning beginning to be of concern at Level 5 well beyond Colin’s grade and Mathletics course. A solid understanding of rational and real numbers does not present itself as an issue until Level 4, again, well beyond Colin’s grade and Mathletics course he was undertaking. My successful, as it turns out, demonstration of percentages and their equivalent fractions along with renaming a number after moving it one decimal place to the right to find 10% from which 5% and 20% could be worked out, actually had little do to do with place value and a lot more to do with helping Colin understand the partitioning of numbers and the relationship between rational and real numbers.

Beatrice

Reflection-in-action pointed towards Beatrice’s poor understanding of equivalence as represented by the = sign as being her initial problem with Euler’s formula. It was not until the complexity of the three-dimensional shapes evolved to point where a different strategy to counting was required. Equivalence is seen as a common misunderstanding for some Level 6 students. Beatrice was working on grade 7 Mathletics material but Seimon
(2006) noted problems at Level 6 could be identified much earlier. In this instance my diagnosis was correct and Beatrice self-solved the problem (Table 2).

**Brian**

My reflection-in-action indicated Brian was struggling with place value with respect to real numbers and this was causing him trouble when he had to round up or down. My teacher-fix for Brian was to show him a number-line but it wasn’t until I pointed out place value that Brian had his ‘ah ha!’ moment. Chapin & Johnson (2000, p. 98) discuss real numbers in terms of their place value when explaining the concept of partitioning; the common Level 4 misunderstanding and above the grade 4 Mathletics content Brian is working on (Table 2). In reflection on reflection-in-action I believe Brian understood place value in terms of a Level 2 student when I reminded him of it, but he had not generalised the idea to that of a number line and consequently being able round up or down. In this case my diagnosis and prescription were correct.

**Andrew (a)**

Reflection-in-action showed Andrew was initially attracted to real numbers with a lot of digits but not necessarily a lot of value. This is a Level 2 misunderstanding of place value and it was only after I drew his attention to the difference in face value of 2 tenths as opposed to 1 tenth that he had his ‘ah ha!’ moment. This relates to a misunderstanding of partitioning, a Level 4 misunderstanding. My diagnosis was incorrect but my prescription worked and helped Andrew find a solution.

**Anne**

Reflection-in-action indicated Anne had a Level 2 place value misunderstanding of where to place numbers on a number line in her grade 7 Mathletics course. Reflection on reflection-in-action indicates this is an incorrect diagnosis and what was really happening here was a failure to apply proportional reasoning. Anne self-solved the problem when she recognised that the halfway point between two numbers on a number line could be given a value, even if the value was negative and/or a real number. Proportional reasoning is a
Discussion arising out of the study unrelated to the common misunderstandings in mathematics

Deanne

My reflection-in-action indicated Deanne’s misunderstanding related to non-mathematical thinking whilst this reflection on reflection-in-action carried out after a review of the literature presented in the previous chapter suggests her misunderstanding was not at all non-mathematical, but arose from a misplaced application of a subitising technique. Deanne’s, ‘ah ha!’ moment came about after she was reminded how to skip count in order to work out the total. The intervention was appropriate and successful but my interpretation of Deanne’s misunderstanding was flawed.

Florence

My reflection-in-action showed Florence initially struggled with the precise meaning of words in probability problems. During my reflection-in-action I diagnosed this as a problem of weak literacy. However, the mismatch between mathematical ideas and words is first mentioned in the Level 6 misunderstanding around the generalising of mathematical ideas. This is way beyond the material Florence is dealing with and I would argue that my original diagnosis as presented in my reflection-in-action is the correct one. Some words in mathematics do have precise meanings and sometimes these are slightly different from the generally accepted meanings of the words. In this instance, Florence was able to make the distinction herself and self-solve her problem.

Carol

This student gets two mentions in Table 2 and they are related to the same problem that she was stumped by in Mathletics. The problem was like Florence’s and concerned determining the probability of particular events. The scenarios, and possible answers were presented by the software in words. Reflection-in-action showed that on the first occasion
Carol self-solved her problem and was ready to share her ‘ah ha!’ moment. I had not witnessed the improvement and took her on her word that she was ready to explain how she was able to solve her problem. The event was a complete failure. It turned out that she had used the help function each time and so, was able to answer each question correctly, but without the help; not so good. We spoke about the importance of the wording and she went away and worked on the task again. Carol’s second attempt also failed. She tried to rote learn each scenario but was unable. At the time I reflected Carol had relied on memory alone and did not own the concept or the rules. On reflection on reflection-in-action I do not believe Carol’s approach relied on mathematical methods at all. The problem was presented in words and so she attempted to answer the question with literal thinking.

Andrew (b)

Reflection-in-action showed Andrew used rules to successfully learn how to identify acute, obtuse and right-angled triangles. After some initial problems, Andrew was able to announce that he had cracked it. I sat down with him and watched him answer all 10 questions in the task and it appeared, on the face of it, that he had self-solved the problem. So, what was his ‘ah ha!’ moment? I was astounded to learn that an artefact of the Mathletics program was that certain angles would align with certain choice buttons and Andrew had figured these out. However, he could not differentiate between acute and obtuse angles outside of the Mathletics environment.

Conclusion

Primary school mathematics occurs from Level 1 to Level 4 inclusively and this is the area where I ostensibly teach, but I have children working on grade 7 content in the Mathletics environment (Table 2). The above reflections on reflections-in-action suggest a weakness in diagnosis but a strength in prescription. They show an underdeveloped content knowledge which led to an overreliance on place value as the cause of the misunderstanding. Some common misunderstandings were not part of my lexicon like subitising, partitioning and proportional thinking. It seems apparent to me, based on my small study, that the common misunderstandings in mathematics as described in Chapter 4
do not acknowledge the problems caused through weak literacy or non-mathematical thinking. In the next chapter I draw some conclusions from the research.
CHAPTER 6 – DISCUSSION AND CONCLUSIONS

Introduction

This Chapter is divided into two parts. In the first part I discuss my research design and in the second part I make some conclusions about my research findings.

Research Design

The introduction of the Mathletics program to the students in the present study was a move-testing experiment (Schön 1987, p. 71) in that it was anticipated that the program could deliver appropriate mathematical curriculum to a multi-age group of students in a single-class school. Collecting data involved writing reflection-in-action pieces based upon digitally recorded presentations by the students of their ‘ah ha!’ moment. The ‘ah ha!’ moment described an instance of student success but the point of interest for the teacher and author of the present study was the misconception that had led to that moment. Catching students proved to problematic. Other work competed for time like numeracy profile testing, a mandated assessment process which required one-on-one interviews. These took one and a half terms to complete and made it difficult to rove and ‘catch’ students for the present study. Many of the students had to volunteer. An unintended consequence of telling my students I wanted to catch their ‘ah ha!’ moment was that they were less likely to give up when faced with a problem presented by the Mathletics course they were studying. When they were stumped, they really were stumped, and not just giving up.

At the same time as I was collecting my data my participation in the postgraduate certificate of primary mathematics teaching resulted in a change in teaching practice where there was far less reliance on Mathletics to deliver content. The change included the introduction of a rich maths task to the weekly program. Which meant the children spent less time on Mathletics, and I had less time to complete the mandated profiling mentioned earlier in Chapter 1. At the end of Semester 1 it also became apparent from writing school reports that the specific teaching of mathematics structure was required and so another session, with a specific focus on structure, was inserted into the weekly mathematics program. It was noted that both the rich maths task and the specific teaching lesson lasted longer than one hour.
In describing professional practice Schön (1987, p. 3) uses the metaphor of the high ground overlooking the swamp and suggests that on the high ground there are well-defined problems that can be solved using research-based theory and the application of technical instruments whereas, ‘In the swampy lowland, messy, confusing problems defy technical solutions (p.3)’. In the present study I was conducting an experiment in my own workplace and clearly, my situation got messy. However, the process of digital recording of students’ own words and gestures and reflecting-in-action about what they had said was a systematic and relatively straightforward way to collect data and interpret it immediately (Chapter 3). In the present study I subsequently engaged with the literature (Chapter 4) and reflected on my earlier reflections-in-action (Chapter 5).

**Research findings**

My research question was to ask what are the common mathematical misconceptions surfaced by the students in the present study using Mathletics? As will be seen in the subsequent discussion, I believed student misunderstanding around place value was a common problem; however I was wrong.

**So?**

Six students demonstrated a mathematical misunderstanding of their Mathletics content. Of these two, Beatrice and Anne, self-solved their misunderstanding but only one diagnosis, Beatrice’s, about equivalence, was correct. Four students, Deanne, Colin, Brian and Andrew relied upon teacher intervention to overcome their problem and subsequently discovered their ‘ah ha!’ moment. But of these, only Brian’s diagnosis, about his place value misunderstanding was correct. On three occasions the diagnosis of the conceptual misunderstanding was wrong and ‘place value’ was mentioned each time in the reflection-in-action. In Colin and Andrew’s case the actual issue was partitioning while in Anne’s case the issue was proportional thinking. Deanne’s issue was thought to be non-mathematical but on reflection on reflection-in-action turned out to be the incorrect application of subitising.

In addition to Deanne, three students, Florence, Carol and Andrew (b), were diagnosed, during the reflection-in-action, as having misunderstandings unrelated to
mathematics. In one instance, Florence, was able to self-solve her problem and it turned out to be an issue related the precise meanings of some words in mathematics. My diagnosis and prescription were correct in that her problem was unrelated to mathematical misunderstandings and more about weak literacy. The other two students are quite interesting. Andrew (b) came up with a rule to work out triangle types that worked in the Mathletics environment but nowhere else. He was trying to rely on a procedure not a conceptual understanding. I correctly identified the misunderstanding during my reflection-in-action. Carol’s self-solving effort proved to be quite interesting too and this was observed in my reflection-in-action. Carol was trying to rote-learn her way to a solution through memorisation of the material found in the help function of the software. She was unsuccessful because she did not have a conceptual understanding of the language of probability.

So what?

I conclude I provided each student experiencing difficulty with an appropriate ‘procedural’ fix. However, it is quite clear that my own grasp of mathematical misunderstandings was wanting. My diagnosis were poor and missed the mark on a number of occasions – my only saving grace was that place value misunderstanding, a diagnosis I used often, is quite critical in a numbers of areas of mathematics (Booker et al., 1998, p. 55). For the purposes of this thesis I made use of common misunderstandings in mathematics as the framework for my literature review; probably because I did not fully appreciate or understand what these were myself.

Now what?

By applying to be on the course I suspect I had an inkling that my prior content knowledge around mathematics was below par and this process of reflection on reflection-in-action has confirmed that it is/was. However, the coursework for the PGCPMT has continued and I find myself being continually exposed to a number of big ideas in mathematics. I now believe any reflection-in-action I might write would be far more accurate with respect to understanding the basis of the student’s misunderstanding.
Conclusion

I conclude that Donald Schön reflection on reflection-in-action has much to commend it as a template for the ‘professional’ thesis written by an experienced practitioner. A professional thesis is different to the academic thesis in that the problem is set by the organisation and not from gaps and conundrums in the literature; it privileges professional knowing-in-action and accepts that the research problem may be messy. The rigor in the process of professional reflection comes from being prepared to accept that one may be right or wrong in one’s initial thinking, and that by honest appraisal of one’s practice against the appropriate literature, it is possible to add new content, ideas, values and skills to one’s knowing-in-action; even if just incrementally.

I conclude that Mathletics is a useful way to deliver content in the multi-age and multi-ability classroom and it is possible for the numeracy teacher to provide just ‘procedural’ support to those students and they will get by. But one has to wonder at what cost to the student? Howe et al. (2010) suggest that for teachers to be effective at teaching [mathematics] they must anticipate where the student is going with their maths learning; Howe et al. (2010) used the metaphor of the chain. Chains can be joined together but the quality of the join effects strength of the chain. A ‘procedural’ fix might be likened using a bit of wire; in that the chain appears to look OK but it has no real integrity and cannot be relied upon in the future. A ‘conceptual’ fix is made of much stronger stuff, perhaps even better quality material, than the chain itself, strong enough in fact to allow other chains of mathematical ideas to be built upon that conceptual point.
REFERENCES


